**SWARNANDHRA**

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**19MA3T05-Probability & Statistics**

**QUESTION BANK (P & S)**

 **UNIT-I**

1. Let A,B and C be three events such that P (A) =1/4, P (B) =1/5, P(C)= 1/6 , P (A∩B) =0 , P(B∩C)=0, P(A∩C)=1/8. Determine the probability that at least one of the events occurs.
2. If A and B are any two arbitrary events of the sample space , then Prove that $P\left(A∪B\right)=P\left(A\right)+P\left(B\right)-P\left(A∩B\right)$ or State and prove Addition theorem on probability in two events .
3. Find the probability of drawing a queen, a king and a jack in that order from a pack of cards in three consecutive draws, the cards drawn not being replaced.
4. A class consists of 6 girls and 10 boys. If a committee of 3 is chosen at random from the class, find the probability that (i) 3 boys are selected. (ii) Exactly 2 girls are selected.
5. Two marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue and 15 orange marbles with replacement being made after each drawing. Find the probability that (i) both are white (ii) first is red and second is white.
6. In a certain college, 25%of the students failed in physics, 15%failed in chemistry and 10% in physics and chemistry. A student is selected at random. (i) If the student failed chemistry ,what is the probability that he/she failed in physics? (ii) If the student failed in physics what is the probability that he/she failed in chemistry?
7. In a certain town, 40 have brown hair, 25% have brown eyes and 15% have both brown hair and brown eyes. A person is selected at random from the town (i) If he has brown hair, what is the probability that he has brown eyes also. (ii) If he has brown eyes, determine the probability that he does not have brown hair .
8. State and prove Baye’s theorem.
9. Of the three men, the chance that a politician, a businessman or an academician,will be appointed as a vice-chancellor (V.C) of a university are 0.5, 0.3, and 0.2 respectively. Probability that research is promoted by these persons if they are appointed as V.C is 0.3, 0.7, and 0.8 respectively. (i) Determine the probability that research is promoted. (ii) If research is promoted, what is the probability that V.C is an academician?
10. Three machines produce 70%, 20% and 10% of total number of the items in a factory. The percentage of defective output of these machines is 4%, 3% and 2% respectively. An item selected at random found defective. Find the probability that it is from (i) Machine -I (ii) Machine-II (iii) Machine –III.
11. In a class 2% of boys and 3% of girls are having blue eyes. There are 30% girls in the class. If a student is selected and having blue eyes; what is the probability that the student is a girl?
12. If two events A and B are independent, show that (i) A’ and B’ are independent, (ii) A’ and B are independent, (iii) A and B’ are independent.
13. The probabilities that students A, B, C, D solve a problem are 1/3, 2/5, 1/5 and 1/4 respectively. If all of them try to solve the problem, what is the probability that the problem is solved?
14. A can hit a target 3 times in 5 shots, B 2 times in 5 shots and C three times in 4 shots. All of them fire one shot each simultaneously at the target. What is the probability that (i) 2 shots hit (ii) at least two shots hit?
15. State the axioms of probability. A box contains 3 white balls, 4 red balls and 5 black balls. Two balls are drawn from the urn at random. Find the probability the both of them are of the same color?
16. A random variable X has the following distribution:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  X |  1 |  2 |  3 |  4 |  8 |  9 |
|  P(X) |  K |  3K |  5K |  7K |  9K |  11K |

Determine (i) k (ii) Mean (iii) Variance

17. Let X be the maximum of two numbers that appear when a pair of fair dice is thrown once. Determine the i) Discrete probability function ii) Expectation iii) Variance.

of X.

18. Verify that the following is a distribution function (x) =$\left\{\begin{array}{c}0 , x\leq -a\\\frac{1}{2}\left(\frac{x}{a}+1\right)\\1,x>a\end{array}\right.,-a\leq x\leq a$

19.A continuous random variable X has the distribution function Fx) = $\left\{\begin{array}{c}0 , if x\leq 1\\k\left(x-1\right)^{4}\\0,if x>3\end{array}\right.,if 1<x\leq 3$

 Find (i) k (ii) f(x) (iii) Mean (iv) Variance.

 20. Let X be a continuous random variable with destiny function $f\left(x\right)=\left\{\begin{array}{c}\frac{x}{6}+k, \&0\leq x\leq 3\\0, \&therwise\end{array}\right.$(i) Evaluate k

 (ii) Find P (1$\leq X\leq 2)$.(iii) mean (iv) variance

**UNIT 2**

1. Find the probability that in tossing a fair coin five items , there will appear
2. 3 heads b) 3 tails and 2 heads c) at least 1 head and d) not more than 1 tail.
3. A die is thrown 8 times and it is required to find the probability that 3 will show
4. Exactly 2 times ii) at least 2 times iii) at most one time
5. Assuming that half of the population are consumers of rice. If 8 individuals are taken to test , Find the probability that
6. Two are consumers of rice.
7. at least two are consumers of rice
8. 1$\leq X\leq 4$ are consumers of rice.
9. 20 % of items produced from a factory are defective. Find the probability that in a sample of 5 chosen at random ,
10. None is defective
11. One is defective
12. P( 1< X <4)
13. The probability that John hits a target is 1/ 2. If he fires 6 times, find the probability that he hits the target

 i) Exactly 2 times ii) more than 4 times iii) at least once.

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1. 20% of bolts produced in a factory found to be defective. Find the probability that in a sample of 10 bolts chosen at random, exactly two will be defective by using(a)binomial distribution

 (b) Poisson approximation to binomial distribution.

1. If a Poisson distribution is such that p(x=1)3/2=p(x=3),

Find (i) P(x>1) (ii) P (x≤3) (iii) P (2≤x≤5).

1. If the probability of a defective fuse from a manufacturing unit is 2%, in a box of 200 fuses, find the probability that(a) Exactly 4 fuses are defective(b) More than 3 fuses are defective.
2. Assume that 50% of all engineering students are good in mathematics. Determine the probabilities that among 18 engineering students (i)exactly 10 (ii)at least 10 (iii)at most 8 (iv)at least 2 and at most 9, are good in mathematics.
3. X is normal variate with mean 30 and standard deviation of 5.Find (i) P (27≤X≤35). (ii) P (X≥45).

 Use normal distribution tables.

11. The mean and Standard deviation of a normal distribution are 8 and4 respectively. Find (i) P (5≤X≤10).

 (ii) P (X≥5).

12 In a normal distribution 7% of the items are under 35 and 89% of the items are under 63. What are the mean and standard deviation of the distribution ?

13. The marks obtain in Mathematics by 1000 students is normally distributed with mean78% and standard deviation 11%(i) Determine how many students got marks above 90% (ii) what was the highest mark obtained by the lowest 10% of the students (iii)within what limits did the middle of 90% of the students lie ?

14. The weekly wages of 1000 workers are normally distributed around a mean of Rs 70 and standard deviation of Rs 5 . Estimate the number of workers whose weekly wages will be (i) between Rs70 and Rs 72

 (ii) between 69 and 72 (iii) more than 80.

15. Suppose the weight of 800 male students are normally distributed with mean of 28.8 kg and standard

 deviation of 2.06 kg. Find the number of students whose weights are (i) between 28.4 kg and 30.4 kg

 (ii) more than 31.3 kg

**UNIT 3**

1. A population consists of four numbers 3, 7, 11, 15. Consider all possible samples of size 2 which can be drawn with replacement from this population. Find i) The mean of the population ii) The standard deviation of the population iii) The mean of the sampling distribution of means
2. Determine the mean and standard deviation of the sampling distribution of means for the population 3, 6, 9, 12, 15 with n=3 and the sampling is without replacement.
3. A population consists of five numbers 2,3,6,8 and 11. Consider all possible samples of size 2 which can be drawn with replacement from this population. Find i) The mean of the population ii) The standard deviation of the population iii) The mean of the sampling distribution of means
4. A random sample of size 100 is taken from an infinite population having the mean µ = 76 and the variance $σ^{2}$ = 256. What is the probability that $\overbar{x}$ will be between 75 and 78.
5. A normal population has a mean of 0.1 and standard deviation of 2.1. Find the probability that mean of a sample of size 900 will be negative.
6. Define Null hypothesis and alternative hypothesis.
7. Explain: Critical Value.
8. What do you mean by level of significance?
9. Explain clearly type I and type II errors.
10. Write a detailed note on one- tailed and two-tailed tests
11. The mean lifetime of 100 fluorescent light bulbs produced by a company is computed to be 1570 hours with a standard deviation of 120 hours. If μ is the mean lifetime of all the bulbs produced by the company, test the hypothesis μ = 1600 hours against the alternative hypothesis μ ≠ 1600 hours using a 5% level of significance.
12. In a random sample of 60 workers the average time taken by them to get to work is 33.8 minutes with a standard deviation of 6.1 minutes, can be reject the H0: μ = 32.6 minutes in favor of H1: μ > 32.6 at α = 0.05 level of significance?
13. Samples of students were drawn from two universities and their weights (in kg) and standard deviation are calculated. Make a large sample test to find the significance of the difference between the means

|  |  |  |  |
| --- | --- | --- | --- |
|  | Size of sample | Mean | Standard deviation |
| University A | 400 | 55 | 10 |
| University B | 100 | 57 | 15 |

1. Experience had shown that 20% of manufactured is of the top quality. In one day’s production of 400 articles only 50 are of top quality. Test the hypothesis at 0.05 level.
2. In a referendum submitted to the ‘student body’ at a university, 850 men and 566 women voted. 530 of the men and 304 of the women voted ‘yes’. Does this indicate a significant difference of the opinion on the matter between men and women students?
3. A random sample of size 81 was taken whose variance is 20.25 and mean is 32 construct 95% confidence interval.
4. A test of the breaking strengths of two different types of cables was conducted using samples of

n1 = n2 = 100 pieces of each type of cable.

Cable I Cable II

x1 =1925 x2 = 1905

σ1= 40 σ2 = 30

Do the data provide sufficient evidence to indicate a difference between the mean breaking strengths of the two cables? Use 0.01 level of significance

**UNIT 4**

1. Write any two properties of t- distribution.
2. 5 measurements of ten content of a certain kind of cigarettes yielded 14.5, 14.2, 14.4, 14.3 and 14.6 mg per cigarette .Show that the average tar claimed by the manufacturer µ=14.0 is significant at $α$=0.05. Assume normality.
3. The average breaking strength of steel rods is specified to be 18.5 thousand pounds . To test this , a sample of 14 rods was tested. The mean and standard deviations obtained were 17.85 and 1.955 respectively. Is the result of the experiment significant.
4. The following are the samples of skills .Test the significant difference between the means at 0.05 level.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| sample1 | 74.1 | 77.7 | 74.4 | 74 | 73.8 |  |
| sample2 | 70.8 | 74.9 | 74.2 | 70.4 | 69.2 | 72.2 |

1. To examine the hypothesis that the husbands are more intelligent than the wives ,an investigator took a sample of 10 couples and administered them a test which measures the IQ as follows

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Husbands | 117  | 105 | 97 | 105 | 123 | 109 | 86 | 78 | 103 | 107 |
| wives | 106 | 98 | 87 | 104 | 116 | 95 | 90 | 69 | 108 | 85 |

 Test the hypothesis with a reasonable test at the level of significance 0.05.

1. The following are the average weekly losses of worker hours due to accidents in 10 industrial plants before and after a certain safety programme was put into operation

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Before  | 45 | 73 | 46 | 124 | 33 | 57 | 83 | 34 | 26 | 17 |
| After | 36 | 60 | 44 | 119 | 35 | 51 | 77 | 29 | 24 | 11 |

 Test whether the safety programme is effective in reducing the number of accidents at the level of significance of 0.05 ?

1. Find 95% confidence limits for the mean of a normally distributed population from which the following sample was taken: 15,17, 10,18, 16, 9, 7, 11, 13, 14.
2. The measurements of the output of two units gave the following results. Assuming that both samples have been obtained from normal population, test at 5% significance level whether the two populations have the same variance.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Unit A | 14.1 | 10.1 | 14.7 | 13.7 | 14.0 |
| Unit B | 14.0 | 14.5 | 13.7 | 12.7 | 14.1 |

1. Write the properties of Chi-square distribution.
2. Fit a Poisson distribution to the following data and test the goodness of fit

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | 0 | 1 | 2 | 3 | 4 |
|  f | 214 | 92 | 20 | 3 | 1 |

1. 4 coins were tossed 160 times and the following results were obtained.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| No. of heads | 0 | 1 | 2 | 3 | 4 |
| Observed frequencies | 17 | 52 | 54 | 31 | 6 |

 Under the assumption that the coins are balanced find the expected frequencies and test goodness of fit at α=0.05.

1. The following table gives number of accidents that occur during the various days

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Days | Sun | Mon | Tue | Wed | Thu | Fri | Sat |
| No. of accidents | 14 | 16 | 8 | 12 | 11 | 9 | 14 |

 Find whether the accidents are uniformly distributed over the week.

1. In an investigation on the machine performance the following results were obtained.

|  |  |  |
| --- | --- | --- |
|  | No. of units inspected | No. of defectives |
| Machine I | 375 | 17 |
| Machine II | 450 | 22 |

Test whether there is any significant performance difference of two machines at α=0.05.

1. The results of polls conducted 2 weeks and 4 weeks before a gubernatorial election are shown in the following table.

|  |  |  |
| --- | --- | --- |
|  | 2 weeks before election | 4 weeks before election |
| For republican candidate | 79 | 91 |
| For Democratic Candidate | 84 | 66 |
|  Undecided | 37 | 43 |

 Use the 0.05 level of significance to test whether there has been a change in opinion during the 2 weeks between the polls.

1. Four methods are under development for making discs of a super conducting material. 50 discs are made by each method and they are checked for super conductivity when cooled with liquid.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1st method  | 2nd method | 3rd method | 4th method |
| Super conductors | 31 | 42 | 22 | 25 |
| Failures | 19 | 8 | 28 | 25 |

 Test the significant difference between the proportions of super conductors at 0.05 level.

1. A large electronic firm that hires many workers with disabilities wants to determine whether their disabilities affect such workers performance. Use the level of significance α=0.05 to decide on the basis of the following data, whether it is reasonable to maintain that the disabilities have no effect on the workers performance.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Above average | Average | Below average |
| Blind | 21 | 64 | 17 |
| Deaf | 16 | 49 | 14 |
| No disability | 29 | 93 | 28 |

1. From the following data find whether here is any significant liking in the habit of taking soft drinks among the categories of employees

|  |  |
| --- | --- |
|  |  Employees |
| Clerks | Teachers | Officers |
| Pepsi | 10 | 25 | 65 |
| Thump Up  | 15 | 30 | 65 |
| Fanta | 50 | 60 | 30 |

1. Mechanical engineers ,Testing a new arc welding technique, classified welds both with respect to appearance and an X-ray inspection

|  |  |  |
| --- | --- | --- |
| X-ray |  |  Quality |
|  | Bad | Normal | Good | Total |
| Bad | 20 | 7 | 3 | 30 |
| Normal | 13 | 51 | 16 | 80 |
| Good | 7 | 12 | 21 | 40 |
| Total | 40 | 70 | 40 | 150 |
|  |  |  |  |  |

**UNIT5**

1. Explain the Principle of least squares.
2. Derive normal equations for fitting (i)a straight line (ii) a parabola (iii) exponential curves of the form y=a ebx and y=a bx (iv) power curve
3. Fit a straight to the form y = a + bx for the following data and estimate the value of y when x = 25

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | 0 | 5 | 10 | 15 | 20 | 25 |
| y | 12 | 15 | 17 | 22 | 24 | 30 |

1. The Fit a parabola by taking x as independent variable.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| y | 1.1 | 1.3 | 1.6 | 2.0 | 2.7 | 3.4 | 4.1 |

1. Fit a curve y = a + bx + cx2 for the following data.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 | 5 |
| y | 6 | 8 | 10 | 12 | 14 |

1. Find the best fit of the type $y=ae^{bx}$ to the following data by the method of least squares.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | 0.0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 |
| y | 0.10 | 0.45 | 2.15 | 9.15 | 40.35 | 180.75 |

1. Using method of least squares fit a relation of the form $y=ab^{x}$ to the following data.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | 2 | 3 | 4 | 5 | 6 |
| y | 144 | 172.3 | 207.4 | 248.8 | 298.5 |

1. Fit a power curve of the form y = axb by the method of least squares for the following data and estimate y at x = 12.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | 20 | 16 | 10 | 11 | 14 |
| y | 22 | 41 | 120 | 89 | 56 |

1. Define correlation and give one example.
2. Define various types of correlation.
3. Define two regression lines
4. Calculate the coefficient of correlation for the following data

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| Y | 15 | 16 | 14 | 13 | 11 | 12 | 10 | 8 | 9 |

 (Ans . 0.95)

1. The following are the marks obtained by 12 students in MEFA and STATISTICS

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| MEFA(X) | 78 | 56 | 36 | 66 | 25 | 75 | 82 | 62 |
| STATISTICS(Y) | 84 | 44 | 51 | 58 | 60 | 68 | 62 | 58 |

Compute the Spearman rank correlation coefficient between MEFA and STATISTICS

 (Ans. 0.6548)

1. Calculate Spearman rank correlation coefficient for the following data

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X | 53 | 98 | 95 | 81 | 75 | 61 | 59 | 55 |
| Y | 47 | 25 | 32 | 37 | 30 | 40 | 39 | 45 |

 (Ans. -0.905)

1. A sample of 12 fathers and their eldest sons gave the following data about their height in inches. Calculate the coefficient of rank correlation

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Fathers | 65 | 63 | 67 | 64 | 68 | 62 | 70 | 66 | 68 | 67 | 69 | 71 |
| Sons | 68 | 66 | 68 | 65 | 69 | 66 | 68 | 65 | 71 | 67 | 68 | 70 |

 (Ans. 0.722)

1. Three judges award the ranks to 8 candidates in an essay competition in the given order

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Judge A | 4 | 3 | 7 | 1 | 6 | 2 | 5 | 8 |
| Judge B | 3 | 5 | 8 | 2 | 7 | 1 | 4 | 6 |
| Judge C | 1 | 4 | 6 | 2 | 8 | 3 | 5 | 7 |

 Using the rank correlation coefficient , determine which pair of judges have common decision making attribute. (Ans. rAB = 0.833 ; rBC =0.8095; rAC =0.7857)

1. Find regression lines for the following data

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | 6 | 2 | 10 | 4 | 8 |
| Y | 9 | 11 | 5 | 8 | 7 |

 ( Ans. X=16.4-1.3Y and Y=11.9-0.65X )

1. Calculate the regression equations taking deviation of items from the mean of X and Y series for the following data

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | 6 | 2 | 10 | 4 | 8 |
| Y | 9 | 11 | 5 | 8 | 7 |

 ( Ans. X=16.4-1.3Y and Y=11.9-0.65X )

1. A chemical company wishing to study the effect of extraction time on the efficiency of an extraction operation obtained in the data shown in the following table

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Extraction time (X) | 27 | 45 | 41 | 19 | 35 | 39 | 19 | 49 | 15 | 31 |
| Extraction efficiency(Y) | 57 | 64 | 80 | 46 | 62 | 72 | 52 | 77 | 57 | 68 |

 Calculate (i) coefficient of correlation between X and Y and (ii) two lines of regression.

(Ans. (i) 0.8248 ,(ii) X=0.8904Y-56.5404 ; Y=0.764X+24.448)